Appendix for Modeling Market Downside Volatility

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1 Derivation of the New Risk-Reward Relationship

Routledge and Zin (2010) derives the stochastic discount factor in a representative agent economy where the investor has recursive utility

$$V_t = \left\{ (1-\delta)\, C_t^{1-\frac{1}{\psi}} + \delta \left[\mathcal{R}_t \left(V_{t+1} \right) \right]^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}}, \tag{1}$$

with the certainty equivalent

$$\frac{\mathcal{R}^{1-\gamma}-1}{1-\gamma} = \int_{-\infty}^{\infty} \frac{V^{1-\gamma}-1}{1-\gamma} dF(V) - \left(\frac{1}{\alpha} - 1 \right) \int_{-\infty}^{\kappa\mathcal{R}} \left(\frac{(\kappa\mathcal{R})^{1-\gamma}-1}{1-\gamma} - \frac{V^{1-\gamma}-1}{1-\gamma} \right) dF(V), \tag{2}$$

where γ is the relative risk aversion parameter, ψ is the elasticity of intertemporal substitution, α is the disappointment aversion parameter and κ is the generalized disappointment aversion parameter. They show that the stochastic discount factor is given by:

$$S_{t,t+1} = \frac{Z_{t+1}^{1-\gamma}}{R_{t+1}} \frac{1 + (1/\alpha - 1)\, I\left(Z_{t+1} < \kappa\right)}{1 + (1/\alpha - 1)\, \kappa^{1-\gamma} \mathbb{E}_t \left[I \left(Z_{t+1} < \kappa \right) \right]} \tag{3}$$

where R_{t+1} is the market return, $I(\cdot)$ denotes the indicator function and where

$$Z_{t+1} = \left(\delta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} R_{t+1} \right)^{\frac{1}{1-\frac{1}{\psi}}}. \tag{4}$$

We assume $\psi = \infty$ and $\kappa = 1$. In this case, we have $Z_{t+1} = \delta R_{t+1}$, and the stochastic discount factor becomes

$$S_{t,t+1} = \delta \left(\delta R_{t+1} \right)^{-\gamma} \frac{1 + (1/\alpha - 1)\, I\left(\delta R_{t+1} < 1\right)}{1 + (1/\alpha - 1)\, \mathbb{E}_t \left[I \left(\delta R_{t+1} < 1 \right) \right]}. \tag{5}$$

Multiplying both numerator and denominator by α and given that

$$I\left(\delta R_{t+1} < 1\right) + I\left(\delta R_{t+1} \geq 1\right) = 1,$$

the stochastic discount factor becomes

$$S_{t,t+1} = \delta \left(\delta R_{t+1} \right)^{-\gamma} \left(\frac{I\left(\delta R_{t+1} < 1\right) + \alpha I\left(\delta R_{t+1} \geq 1\right)}{\mathbb{E}_t \left[I\left(\delta R_{t+1} < 1\right) \right] + \alpha \mathbb{E}_t \left[I\left(\delta R_{t+1} \geq 1\right) \right]} \right). \tag{6}$$

We also have that

$$\begin{aligned} S_{t,t+1} &= \delta \left(\delta R_{t+1} \right)^{-\gamma} \frac{1 + (\alpha - 1)\, I\left(\delta R_{t+1} \geq 1\right)}{1 + (\alpha - 1)\, \mathbb{E}_t \left[I\left(\delta R_{t+1} \geq 1\right) \right]} \\ &= \delta^{1-\gamma} \frac{\exp\left(-\gamma r_{t+1}\right) + (\alpha - 1) \exp\left(-\gamma r_{t+1}\right) I\left(r_{t+1} \geq -\ln \delta\right)}{1 + (\alpha - 1)\, \mathbb{E}_t \left[I\left(r_{t+1} \geq -\ln \delta\right) \right]}. \end{aligned} \tag{7}$$

We also have that

$$
\begin{aligned}
S_{t,t+1}R_{t+1} &= \delta\left(\delta R_{t+1}\right)^{1-\gamma}\frac{1+(\alpha-1)\,I\left(\delta R_{t+1}\geq 1\right)}{1+(\alpha-1)\,\mathbb{E}_t\left[I\left(\delta R_{t+1}\geq 1\right)\right]} \\
&= \delta^{1-\gamma}\frac{\exp\left(\left(1-\gamma\right)r_{t+1}\right)+(\alpha-1)\exp\left(\left(1-\gamma\right)r_{t+1}\right)I\left(r_{t+1}\geq-\ln\delta\right)}{1+(\alpha-1)\,\mathbb{E}_t\left[I\left(r_{t+1}\geq-\ln\delta\right)\right]}.
\end{aligned} \tag{8}
$$

Let $M_t\left(u\right)=\mathbb{E}_t\left[\exp\left(ur_{t+1}\right)\right]$ and $M_t\left(u;x\right)=\mathbb{E}_t\left[\exp\left(ur_{t+1}\right)I\left(r_{t+1}\geq x\right)\right]$ denote the conditional moment generating function and the conditional truncated moment generating function of returns r_{t+1}. Taking the conditional expectation of (8), we have

$$
\begin{aligned}
\mathbb{E}_t\left[S_{t,t+1}R_{t+1}\right] &= \delta^{1-\gamma}\frac{\mathbb{E}_t\left[\exp\left(\left(1-\gamma\right)r_{t+1}\right)\right]+(\alpha-1)\,\mathbb{E}_t\left[\exp\left(\left(1-\gamma\right)r_{t+1}\right)I\left(r_{t+1}\geq-\ln\delta\right)\right]}{1+(\alpha-1)\,\mathbb{E}_t\left[I\left(r_{t+1}\geq-\ln\delta\right)\right]} \\
&= \delta^{1-\gamma}\frac{M_t\left(1-\gamma\right)+(\alpha-1)\,M_t\left(1-\gamma;-\ln\delta\right)}{1+(\alpha-1)\,M_t\left(0;-\ln\delta\right)}.
\end{aligned} \tag{9}
$$

Assuming that log returns are conditionally binormally distributed, we need the conditional moment generating function $M_t\left(u\right)=\mathbb{E}_t\left[\exp\left(ur_{t+1}\right)\right]$ as well as the conditional truncated moment generating function $M_t\left(u;x\right)=\mathbb{E}_t\left[\exp\left(ur_{t+1}\right)I\left(r_{t+1}\geq x\right)\right]$ of returns to be able to explicitly expressed the Euler equilibrium restriction $\mathbb{E}_t\left[S_{t,t+1}R_{t+1}\right]=1$. These functions are given by:

$$
M_t\left(u\right)=\frac{2\sigma_{1,t}}{\sigma_{1,t}+\sigma_{2,t}}\exp\left(m_tu+\frac{\sigma_{1,t}^2u^2}{2}\right)\Phi\left(-\sigma_{1,t}u\right)+\frac{2\sigma_{2,t}}{\sigma_1+\sigma_{2,t}}\exp\left(m_tu+\frac{\sigma_{2,t}^2u^2}{2}\right)\Phi\left(\sigma_{2,t}u\right) \tag{10}
$$

and

$$
M_t\left(u;x\right)=M_t\left(u\right)-\frac{2\sigma_{1,t}}{\sigma_{1,t}+\sigma_{2,t}}\exp\left(m_tu+\frac{\sigma_{1,t}^2u^2}{2}\right)\Phi\left(\frac{x-m_t}{\sigma_{1,t}}-\sigma_{1,t}u\right)\quad\text{if}\quad x<m_t \tag{11}
$$

$$
=\frac{2\sigma_{2,t}}{\sigma_{1,t}+\sigma_{2,t}}\exp\left(m_tu+\frac{\sigma_{2,t}^2u^2}{2}\right)\Phi\left(-\frac{x-m_t}{\sigma_{2,t}}+\sigma_{2,t}u\right)\quad\text{if}\quad x\geq m_t, \tag{12}
$$

where Φ is the standard normal cumulative distribution function.

Since the Euler equilibrium condition implies that $\mathbb{E}_t\left[S_{t,t+1}R_{t+1}\right]=1$, then, substracting 1 from each side of equation (9) yields

$$
0=G\left(m_t,\sigma_{1,t},\sigma_{2,t}\right), \tag{13}
$$

where

$$G\left(m_t,\sigma_{1,t},\sigma_{2,t}\right)=\delta^{1-\gamma}\frac{M_t\left(1-\gamma\right)+\left(\alpha-1\right)M_t\left(1-\gamma;-\ln\delta\right)}{1+\left(\alpha-1\right)M_t\left(0;-\ln\delta\right)}-1. \tag{14}$$

The moment generating function of $r_{t+1}+\ln\delta$ is

$$\tilde{M}_t\left(u\right)=\mathbb{E}_t\left[\exp\left(u\left(r_{t+1}+\ln\delta\right)\right)\right]=\delta^u M_t\left(u\right),$$

and its truncated moment generating function is

$$\tilde{M}_t\left(u;x\right)=\mathbb{E}_t\left[\exp\left(u\left(r_{t+1}+\ln\delta\right)\right)I\left(r_{t+1}+\ln\delta\geq x\right)\right]=\delta^u M_t\left(u;x-\ln\delta\right).$$

Applying the two functions at $u=1-\gamma$ and $x=0$, it turns out that

$$G\left(m_t,\sigma_{1,t},\sigma_{2,t}\right)=\frac{\tilde{M}_t\left(1-\gamma\right)+\left(\alpha-1\right)\tilde{M}_t\left(1-\gamma;0\right)}{1+\left(\alpha-1\right)\tilde{M}_t\left(0;0\right)}-1. \tag{15}$$

Given that at the steady state we have $G\left(\bar{m},\bar{\sigma}_1,\bar{\sigma}_2\right)=0$, applying the implicit theorem function to equation (13), there exists a unique continuous and derivable function g so that

$$m_t=-\ln\delta+g\left(\sigma_{1,t},\sigma_{2,t}\right)$$

in the neighborhood of $\left(\bar{\sigma}_1,\bar{\sigma}_2\right)$. Moreover, the partial derivatives of this function with respect to its two arguments are given by;

$$g_{\sigma_1}=\frac{\partial g}{\partial\sigma_1}=\frac{\partial G/\partial\sigma_1}{\partial G/\partial m}=\frac{G_{\sigma_1}}{G_m}\ \text{ and }\ g_{\sigma_2}=\frac{\partial g}{\partial\sigma_2}=\frac{\partial G/\partial\sigma_2}{\partial G/\partial m}=\frac{G_{\sigma_2}}{G_m}.$$

Finally, it follows that the first-order linearization of the conditional mode m_t in the neighborhood of $\left(\bar{\sigma}_1,\bar{\sigma}_2\right)$ is given by

$$m_t=-\ln\delta+g\left(\sigma_{1,t},\sigma_{2,t}\right)\approx\lambda_0+\lambda_1\sigma_{1,t}+\lambda_2\sigma_{2,t}, \tag{16}$$

where

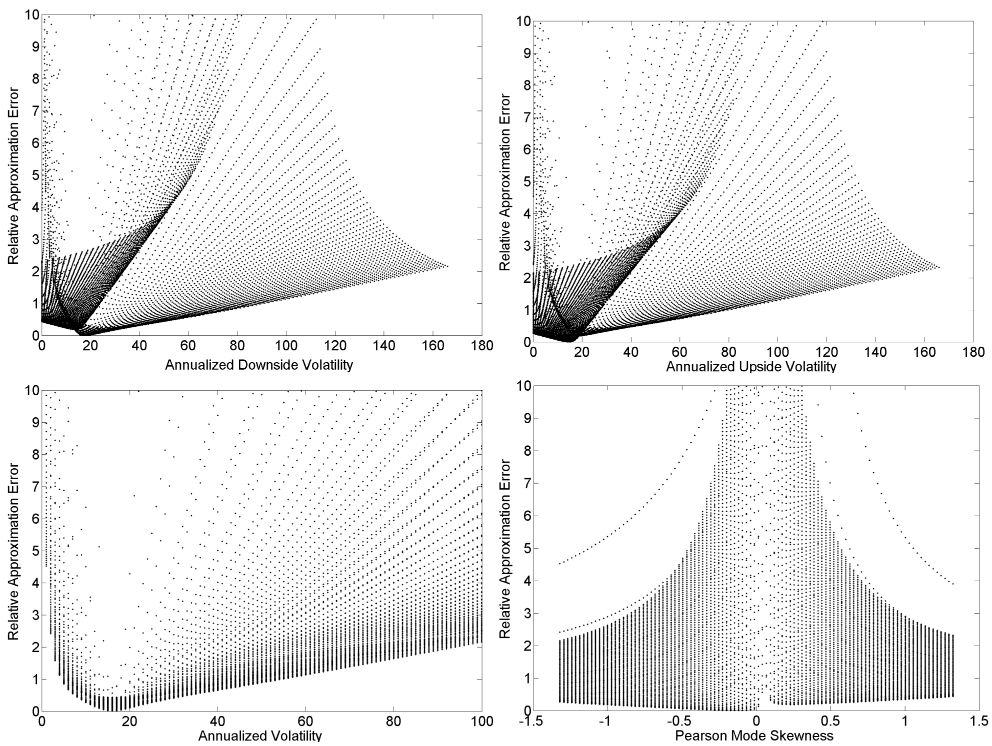
$$\begin{aligned}\lambda_1 &= -\frac{G_{\sigma_1}\left(\bar{m},\bar{\sigma}_1,\bar{\sigma}_2\right)}{G_m\left(\bar{m},\bar{\sigma}_1,\bar{\sigma}_2\right)},\ \lambda_2=-\frac{G_{\sigma_2}\left(\bar{m},\bar{\sigma}_1,\bar{\sigma}_2\right)}{G_m\left(\bar{m},\bar{\sigma}_1,\bar{\sigma}_2\right)} \\ &\quad\text{and } \lambda_0=\bar{m}-\lambda_1\bar{\sigma}_1-\lambda_2\bar{\sigma}_2,\end{aligned} \tag{17}$$

and where $\bar{m}$ satisfies $G\left(\bar{m},\bar{\sigma}_1,\bar{\sigma}_2\right)=0$.

Since we linearize equation (13) around the steady state values $(\bar{\sigma}_1, \bar{\sigma}_2)$, it is only rational to examine the approximation errors generated by the linearization process. Figure 1 shows the approximation errors vis-à-vis upside, downside, and total volatility, as well as Pearson mode skewness. We generated these plots based on values of preference parameter, α and γ, estimated from full sample of S&P500 returns. We conduct a grid search on a 10,000-points grid generated from reasonable volatility and Pearson mode skewness values. We compare the numerically solved true values for conditional mode with the linearized approximations. We find that 8.5% of computed volatility-Pearson mode skewness couples, have approximation errors that exceed 10%. The same couples also correspond to values of upside and downside volatility that are significantly different from the steady state values of 17% and 15%. Thus, we conclude that our linearization of equation (15) is reasonably accurate.

2 Additional Tables and Figures

Figure 1: **Approximation Errors in Linearization of Non-Linear Risk-Return Relationship**



This figure plots the approximation error of linearizing equation (10) as a function of volatility, Pearson mode skewness, downside volatility and upside volatility. The plots in this figure are based on sample average values of volatility and Pearson mode skewness estimated for the S&P500 daily returns.

Table 1: **ML Estimation of Full, Unrestricted BiN-GARCH Models for International Daily Excess Returns.**

Country	Parameter	1980-1988	1990-1999	2000-2009	1980-2009	Country	Parameter	1980-1988	1990-1999	2000-2009	1980-2009
Denmark	λ_0	0.0005	0.0001	0.0005	0.0004	Italy	λ_0	−0.0074∗	−0.0017	0.0004	−0.0007
		(0.0014)	(0.0008)	(0.0008)	(0.0005)			(0.0031)	(0.0011)	(0.0009)	(0.0005)
	λ_1	0.1020	0.4406∗	−0.0936	0.5185∗		λ_1	−0.2164	0.5329∗	1.2561∗	0.4977∗
		(0.3215)	(0.0960)	(1.3712)	(0.1353)			(1.1374)	(0.0513)	(0.5468)	(0.0762)
	λ_2	−0.1163	−0.3910∗	0.2788	−0.5103∗		λ_2	0.9397	−0.3799∗	−1.4283	−0.3968∗
		(0.3113)	(0.1057)	(1.6874)	(0.1553)			(1.3735)	(0.0804)	(0.6581)	(0.0858)
Log Lik		7063.74	9173.18	7779.92	23962.25			6561.68	8442.13	7827.45	22761.00
BIC		-3.0236	-3.1221	-2.9488	-3.0489			-2.8061	-2.8709	-2.9670	-2.8954
France	λ_0	0.0014	−0.0029∗	0.0001	−0.0002	Netherlands	λ_0	0.0013	0.0005	$-2.47E-05$	0.0004
		(0.0014)	(0.0007)	(0.0006)	(0.0005)			(0.0009)	(0.0007)	(0.0006)	(0.0004)
	λ_1	−0.9613	0.6768∗	0.8956∗	0.3843∗		λ_1	0.8987∗	0.9622∗	0.6594∗	0.8838∗
		(1.0486)	(0.0514)	(0.1253)	(0.1710)			(0.1159)	(0.1701)	(0.1989)	(0.0919)
	λ_2	1.1022	−0.3349∗	−0.9393∗	−0.2837		λ_2	−1.0174∗	−1.0177∗	−0.6430∗	−0.9239∗
		(1.1528)	(0.0803)	(0.1707)	(0.2008)			(0.1359)	(0.2377)	(0.2431)	(0.1188)
Log Lik		6905.78	9102.22	7621.80	23575.36			6921.49	9582.42	7750.16	24223.90
BIC		-2.9552	-3.0978	-2.8882	-2.9995			-2.9620	-3.2628	-2.9374	-3.0823
Greece	λ_0		−0.0011	0.0019∗	0.0003	New Zealand	λ_0		$8.17555E-05$	0.0015	0.0012
			(0.0008)	(0.0009)	(0.0006)				(0.0011)	(0.0009)	(0.0007)
	λ_1		0.4271∗	0.1902	0.2604∗		λ_1		0.4597∗	0.6285∗	0.3441∗
			(0.1185)	(0.1784)	(0.1234)				(0.0928)	(0.1132)	(0.1438)
	λ_2		−0.3826∗	−0.3171‡	−0.3027∗		λ_2		−0.4833∗	−0.7117∗	−0.4074∗
			(0.0915)	(0.1874)	(0.1114)				(0.0930)	(0.1639)	(0.1458)
Log Lik			7661.47	7225.52	15491.94				8572.27	7636.36	16753.38
BIC			-2.6027	-2.7363	-2.6856				-2.9156	-2.8938	-2.9057
Hong Kong	λ_0	0.0016	0.0013	0.0003	0.0009∗	Norway	λ_0	0.0010	0.0025∗	0.0004	0.0014∗
		(0.0011)	(0.0008)	(0.0006)	(0.0005)			(0.0020)	(0.0012)	(0.0011)	(0.0006)
	λ_1	0.3142∗	0.3500∗	0.4424∗	0.3344∗		λ_1	0.3709∗	−0.1131	0.5800∗	0.3371∗
		(0.1041)	(0.0746)	(0.1003)	(0.0606)			(0.1440)	(0.2769)	(0.1670)	(0.1085)
	λ_2	−0.2712∗	−0.3984∗	−0.4366∗	−0.3259∗		λ_2	−0.4539∗	−0.0203	−0.5274∗	−0.3970∗
		(0.1344)	(0.0754)	(0.1110)	(0.0674)			(0.1202)	(0.2825)	(0.2413)	(0.1158)
Log Lik		6151.93	8247.50	7720.01	22060.55			6567.06	8608.95	7216.17	22349.92
BIC		-2.6286	-2.8040	-2.9258	-2.8059			-2.8084	-2.9283	-2.7327	-2.8429
Ireland	λ_0		$-2.83E-05$	0.0011	0.0006	Singapore	λ_0	0.0002	−0.0006	0.0010‡	$2.42E-05$
			(0.0008)	(0.0008)	(0.0007)			(0.0013)	(0.0011)	(0.0006)	(0.0004)
	λ_1		0.6511∗	0.9513∗	0.5595∗		λ_1	0.2184‡	0.1601	0.6781∗	0.2433∗
			(0.1018)	(0.2198)	(0.1560)			(0.1250)	(0.1267)	(0.0655)	(0.0705)
	λ_2		−0.6289∗	−1.1103∗	−0.5837∗		λ_2	−0.1814‡	−0.0446	−0.7430∗	−0.1820∗
			(0.1265)	(0.2632)	(0.2132)			(0.0986)	(0.1323)	(0.0857)	(0.0766)
Log Lik			9006.68	7380.60	17027.26			7081.85	9167.96	7757.16	23948.82
BIC			-3.0649	-2.7957	-2.9534			-3.0315	-3.1204	-2.9401	-3.0472

This table reports maximum likelihood estimation results of the BiN-GARCH model for ten international markets' daily excess returns. The sample includes continuously compounded value-weighted returns on country indexes starting on January 1, 1980 and ending in December 31, 2009. Standard errors are reported below the estimated parameters. BIC represents Bayesian information criteria. * and ‡ indicate statistical significance of the estimated parameters at 5% and 10% levels, respectively. λ_i are as in $m_t = \lambda_0 + \lambda_1\sigma_{1,t} + \lambda_2\sigma_{2,t}$. Unreported parameters are due to different start dates for data series. Source: Thomson Reuters Datastream.

Table 2: **Comparison between NGARCH with Binormal and Skewed Student's-t Distributions: NGARCH with Skewed Student's-t Errors.**

Index	Parameter	1980-1989	1990-1999	2000-2009	1980-2009	Index	Parameter	1980-1989	1990-1999	2000-2009	1980-2009
S&P500	β_0	$2.9E-06$*	$5.8E-07$*	$1.3E-06$*	$1.0E-06$*	Germany	β_0	$5.2E-06$*	$1.7E-06$*	$3.4E-06$*	$2.7E-06$*
		($8.3E-07$)	($2.2E-07$)	($2.5E-07$)	($1.8E-07$)			($1.6E-06$)	($5.8E-07$)	($6.3E-07$)	($4.4E-07$)
	β_1	0.9182*	0.9175*	0.8346*	0.9043*		β_1	0.8807*	0.9339*	0.8690*	0.9024*
		(0.0179)	(0.0161)	(0.0190)	(0.0090)			(0.0214)	(0.0120)	(0.0138)	(0.0081)
	β_2	0.0405*	0.0530*	0.0517*	0.0555*		β_2	0.0805*	0.0506*	0.0706*	0.0701*
		(0.0087)	(0.0097)	(0.0078)	(0.0052)			(0.0142)	(0.0093)	(0.0092)	(0.0062)
	θ	0.4573*	0.6748*	1.4489*	0.7729*		θ	0.2394*	0.2830*	0.8349*	0.4357*
		(0.1818)	(0.1336)	(0.1838)	(0.0791)			(0.0962)	(0.1255)	(0.1196)	(0.0604)
	η	5.8773*	6.7339*	14.4991*	7.2620*		η	11.5476*	6.5704*	21.2475*	9.4010*
		(0.6621)	(0.8586)	(3.6724)	(0.5761)			(2.4159)	(0.7098)	(8.0001)	(0.8543)
	λ	−0.0079	−0.0322	−0.1191*	−0.0523*		λ	0.0106	−0.0974*	−0.0896*	−0.0635*
		(0.0317)	(0.0263)	(0.0264)	(0.0149)			(0.0265)	(0.0252)	(0.0276)	(0.0155)
	s	−0.0281	−0.1009	−0.2487*	−0.1540*		s	0.0243	−0.3099*	−0.1712*	−0.1587*
		(0.1132)	(0.0816)	(0.0564)	(0.0443)			(0.0606)	(0.0829)	(0.0540)	(0.0393)
	k	6.1971*	5.2069*	3.6258*	4.8664*		k	3.7955*	5.4527*	3.3719*	4.1360*
		(1.1270)	(0.6907)	(0.2112)	(0.3291)			(0.2546)	(0.6780)	(0.1671)	(0.1794)
Likelihood		8308.39	8687.34	7890.52	24837.79			6988.02	8968.01	7452.71	23374.03
BIC		-3.2674	-3.4161	-3.1143	-3.2733			-3.0043	-3.0626	-2.8354	-2.9783
Australia	β_0	$1.7E-05$*	$1.0E-05$*	$3.9E-06$*	$6.0E-06$*	U.K.	β_0	$9.5E-06$*	$7.3E-07$*	$2.4E-06$*	$1.9E-06$*
		($4.3E-06$)	($2.9E-06$)	($8.2E-07$)	($9.7E-07$)			($3.8E-06$)	($3.5E-07$)	($4.6E-07$)	($3.6E-07$)
	β_1	0.8120*	0.8198*	0.8787*	0.8748*		β_1	0.8571*	0.9553*	0.8534*	0.9098*
		(0.0367)	(0.0390)	(0.0179)	(0.0140)			(0.0373)	(0.0117)	(0.0155)	(0.0082)
	β_2	0.0961*	0.0684*	0.0605*	0.0761*		β_2	0.0817*	0.0247*	0.0820*	0.0666*
		(0.0213)	(0.0142)	(0.0112)	(0.0089)			(0.0178)	(0.0070)	(0.0108)	(0.0066)
	θ	0.2242*	0.6513*	0.8197*	0.4148*		θ	0.1916‡	0.6978*	0.8064*	0.4140*
		(0.1068)	(0.1696)	(0.1489)	(0.0653)			(0.1122)	(0.2247)	(0.1112)	(0.0640)
	η	7.1087*	7.5328*	7.8119*	7.1618*		η	11.2445*	9.3359*	22.1561*	10.9179*
		(0.9320)	(0.9877)	(1.0115)	(0.5201)			(2.1171)	(1.3479)	(8.2865)	(1.1326)
	λ	−0.0334	−0.0235	−0.1333*	−0.0558*		λ	−0.1129*	−0.0563*	−0.0799*	−0.0802*
		(0.0296)	(0.0254)	(0.0288)	(0.0157)			(0.0299)	(0.0265)	(0.0276)	(0.0160)
	s	−0.1001	−0.0674	−0.3684*	−0.1661*		s	−0.2583*	−0.1414*	−0.1517*	−0.1864*
		(0.0892)	(0.0727)	(0.0822)	(0.0471)			(0.0699)	(0.0668)	(0.0528)	(0.0376)
	k	4.9417*	4.7034*	4.7225*	4.9294*		k	3.8908*	4.1445*	3.3492*	3.9002*
		(0.5822)	(0.4757)	(0.4549)	(0.3171)			(0.2569)	(0.2882)	(0.1546)	(0.1461)
Likelihood		6733.18	9138.00	7818.09	23652.94			6890.17	9498.65	7986.20	24333.18
BIC		-2.8938	-3.1210	-2.9755	-3.0139			-2.9619	-3.2450	-3.0399	-3.1009

This table reports maximum likelihood estimation results of Hansen (1994) model for S&P500 and three international markets' daily excess returns. The sample starts on January 1, 1980 and ends in December 31, 2009. Standard errors are reported below the estimated parameters. BIC represents Bayesian information criteria. * and ‡ indicate statistical significance of the estimated parameters at 5% and 10% levels, respectively. η, λ, s, and k represent shape parameters for skewness and kurtosis, and implied skewness and kurtosis, respectively.

Table 3: **Comparison between NGARCH with Binormal and Skewed Student's-t Distributions: NGARCH with Binormal Errors.**

Index	Parameter	1980-1989	1990-1999	2000-2009	1980-2009	Index	Parameter	1980-1989	1990-1999	2000-2009	1980-2009
S&P500	λ_0	0.0011* (0.0003)	0.0008* (0.0003)	0.0005‡ (0.0003)	0.0009* (0.0002)	Germany	λ_0	−0.0004 (0.0005)	0.0022* (0.0004)	0.0012* (0.0004)	0.0012* (0.0002)
	β_0	$4.70E-06$* ($1.12E-06$)	$7.17E-07$* ($2.41E-07$)	$1.0E-06$* ($2.6E-07$)	$1.15E-06$* ($2.02E-07$)		β_0	$5.4E-06$* ($1.6E-06$)	$2.4E-06$* ($1.1E-06$)	$2.7E-06$* ($6.1E-07$)	$3.2E-06$* ($5.1E-07$)
	β_1	0.8493* (0.0221)	0.9079* (0.0138)	0.8476* (0.0165)	0.8920* (0.0081)		β_1	0.8728* (0.0211)	0.9073* (0.0225)	0.8737* (0.0123)	0.8878* (0.0087)
	β_2	0.0806* (0.0112)	0.0565* (0.0088)	0.0497* (0.0072)	0.0628* (0.0050)		β_2	0.0888* (0.0144)	0.0755* (0.0175)	0.0710* (0.0090)	0.0835* (0.0071)
	θ	0.5923* (0.1084)	0.7434* (0.1158)	1.4377* (0.1714)	0.8078* (0.0668)		θ	0.2515* (0.0820)	0.3634* (0.0893)	0.8249* (0.1140)	0.4462* (0.0508)
	p	−0.1221* (0.0346)	−0.0937* (0.0332)	−0.0941* (0.0238)	−0.1138* (0.0187)		p	0.0247 (0.0400)	−0.2034* (0.0312)	−0.1039* (0.0330)	−0.1091* (0.0195)
	s	−0.1219* (0.0344)	−0.0935* (0.0331)	−0.0939* (0.0237)	−0.1136* (0.0186)		s	0.0247 (0.0399)	−0.2022* (0.0306)	−0.1038* (0.0329)	−0.1089* (0.0194)
	k	3.0106* (0.0060)	3.0062* (0.0044)	3.0063* (0.0032)	3.0092* (0.0030)		k	3.0004* (0.0014)	3.0290* (0.0088)	3.0077* (0.0049)	3.0084* (0.0030)
Likelihood		8196.30	8632.71	7872.64	24657.87			6971.77	8833.91	7448.00	23227.32
BIC		-3.2262	-3.3976	-3.1104	-3.2507			-3.0006	-3.0193	-2.8366	-2.9607
Australia	λ_0	0.0021* (0.0005)	0.0007‡ (0.0004)	0.0031* (0.0004)	0.0020* (0.0002)	U.K.	λ_0	0.0026* (0.0005)	0.0008* (0.0003)	0.0007* (0.0003)	0.0012* (0.0002)
	β_0	$3.15E-05$* ($5.97E-06$)	$8.7E-06$* ($2.3E-06$)	$3.1E-06$* ($8.7E-07$)	$7.5E-06$* ($1.15E-06$)		β_0	$1.5E-05$* ($4.2E-06$)	$4.4E-07$* ($2.2E-07$)	$1.8E-06$* ($4.3E-07$)	$1.8E-06$* ($3.4E-07$)
	β_1	0.6218* (0.0462)	0.8223* (0.0316)	0.8643* (0.0230)	0.8172* (0.0161)		β_1	0.8157* (0.0379)	0.9647* (0.0080)	0.8585* (0.0139)	0.9099* (0.0076)
	β_2	0.2420* (0.0299)	0.0751* (0.0120)	0.1118* (0.0177)	0.1371* (0.0114)		β_2	0.0860* (0.0156)	0.0216* (0.0051)	0.0832* (0.0104)	0.0653* (0.0061)
	θ	0.1562* (0.0602)	0.6652* (0.1349)	0.4616* (0.0781)	0.3311* (0.0410)		θ	0.3558* (0.1127)	0.6728* (0.2078)	0.8058* (0.1034)	0.4724* (0.0601)
	p	−0.1431* (0.0372)	−0.0721* (0.0348)	−0.2754* (0.0310)	−0.1707* (0.0198)		p	−0.2052* (0.0411)	−0.0708* (0.0351)	−0.0907* (0.0330)	−0.1212* (0.0200)
	s	−0.1427* (0.0369)	−0.0721* (0.0347)	−0.2724* (0.0300)	−0.1700* (0.0196)		s	−0.2040* (0.0403)	−0.0708* (0.0350)	−0.0906* (0.0328)	−0.1209* (0.0199)
	k	3.0145* (0.0075)	3.0037* (0.0036)	3.0526* (0.0115)	3.0206* (0.0047)		k	3.0296* (0.0117)	3.0036* (0.0035)	3.0058* (0.0042)	3.0104* (0.0034)
Likelihood		6652.79	9083.73	7713.79	23382.49			6860.74	9453.31	7981.39	24233.93
BIC		-2.8624	-3.1051	-2.9385	-2.9805			-2.9525	-3.2321	-3.0411	-3.0893

This table reports maximum likelihood estimation results of the NGARCH model of Engle and Ng (1993) for S&P500 and three international markets' daily excess returns. The sample starts on January 1, 1980 and ends in December 31, 2009. Standard errors are reported below the estimated parameters. BIC represents Bayesian information criteria. * and ‡ indicate statistical significance of the estimated parameters at 5% and 10% levels, respectively. p, s, and k represent Pearson mode skewness, skewness, and kurtosis, respectively.