Supplementary Appendix to

Volatility in Equilibrium: Asymmetries and Dynamic Dependencies

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Abstract

This web-based appendix contains additional empirical results, robustness checks, and calibrations that compliment the findings reported in the paper.

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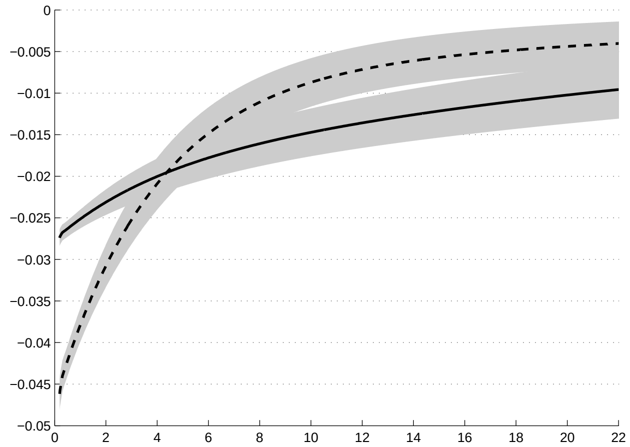
1 Impulse Response Analysis

A reviewer noted that ".. a way to think of the left-side of the cross-correlation diagram between returns and volatility is as an impulse-response function (with time running from right to left)." We agree. To further elaborate on this point, Figure A1 shows the observed orthogonalized VAR impulse response functions for the the variance risk premium and the $\mathbf{VIX}^2$, respectively, to a one standard deviation shock in the return in a system comprised of the return, the variance premium, and the $\mathbf{VIX}^2$, in that order. Figure A2 shows the equivalent impulse responses calculated for the same equilibrium model calibration underlying *Figure 3* of the paper. The observed and model-implied responses are qualitatively similar and importantly of the same general orders of magnitude. Of course, the agreement across the two figures is not perfect, just as the calibrated correlation plots in *Figure 3* of the paper do not provide a perfect fit to the actual cross-correlations in the data.

2 Measurement Error in the Variance Risk Premium

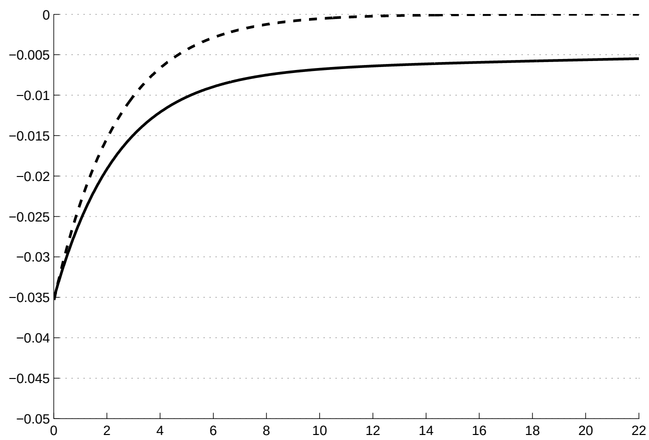
The proxy for the variance risk premium variable used in the paper relies on an estimate for the conditional mean for the integrated variance in place of the population conditional mean, and as such is invariably measured with error. This error-in-variables problem could potentially bias downward the raw sample autocorrelations. Hansen and Lunde (2010) have recently studied this same issue in the context of estimating the autocorrelation functions of realized variation measures, which are similarly affected by estimation errors. Hansen and Lunde (2010) develop a general instrumental variables procedure that renders the estimated autocorrelations robust to this form of measurement error. We implemented their procedure with the results shown in Figure A3, based on short lag lengths for the instruments, and Figure A4, based on long lags for the instruments. In either case, there is little systematic difference between the raw and robustly estimated autocorrelation functions, suggesting that the actual form of the measurement errors in our proxy for the variance risk premium do not materially affect the estimated correlations. To allow for a direct comparison with *Figure 1* of the paper, we also include in Figure A5 the raw and measurement-error robust sample autocorrelations for *both* the premium and the $\mathbf{VIX}^2$, based on an intermediate choice of lagged instruments.

Figure A1 Observed Orthogonalized Impulse Response Functions



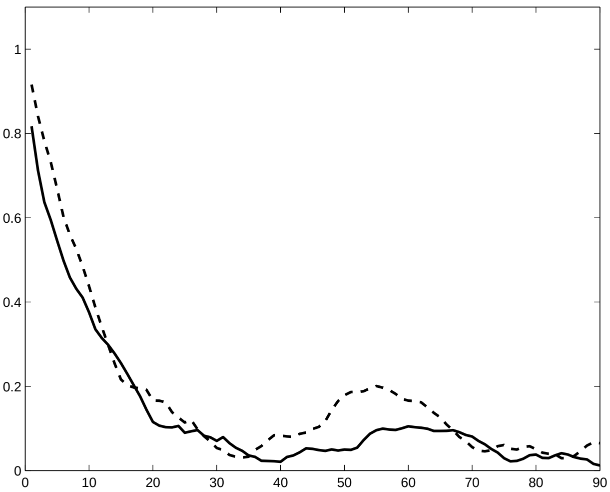
Solid line: response of the $\mathbf{VIX}^2$. Dashed line: response of the variance premium. The shaded areas represent 95 percent confidence intervals from the estimated VARs.

Figure A2 Model-Implied Orthogonalized Impulse Response Functions



Solid line: response of the $\mathbf{VIX}^2$. Dashed line: response of the variance premium.

Figure A3 Raw and Short-Lag Measurement-Error Robust Sample Autocorrelations

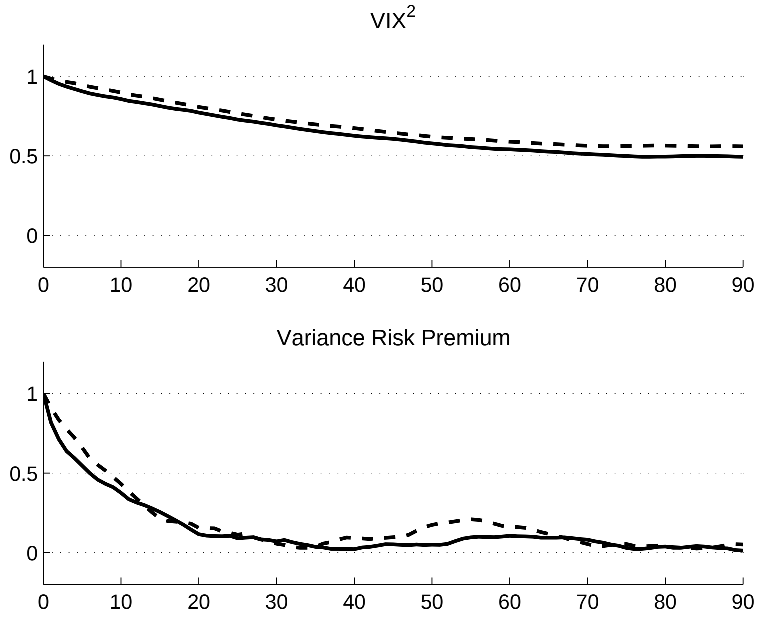


Solid line: raw sample autocorrelation function for the variance risk premium. Dashed line: estimated autocorrelation using the error-robust procedure of Hansen and Lunde (2010) with lags 5–22 (days) used for instruments.

Figure A4 Raw and Long-Lag Measurement-Error Robust Sample Autocorrelations

Solid line: raw sample autocorrelation function for the variance risk premium. Dashed line: estimated autocorrelation using the error-robust procedure of Hansen and Lunde (2010) with lags 22–250 (days) used for instruments.

Figure A5 Raw and Measurement-Error Robust Sample Autocorrelations



Solid lines: raw sample autocorrelation functions. Dashed lines: estimated autocorrelations using the error-robust procedure of Hansen and Lunde (2010) with lags 5–66 (days) used for instruments.

3 Additional Calibration Results

To further illustrate the working of the basic model described in the paper, we undertook two additional calibrations. Table A1 shows the parameter values for each of the calibrations and their sources. To facilitate the calculations, we simply fixed the two log-linearization parameters κ_1 and κ_1^d at $(1/0.997)-1$ and $(1/0.97)-1$ for each of the two calibrations, respectively, rather than solve for these within the model. As pointed out by the referee, $\kappa_1 = \kappa_1^d$ implies that the price-dividend and price-consumption ratios are the same within the calibrations, while the greater risk of the dividend claim naturally implies that the former ratio should be higher. Also, the value of $\kappa_1 = (1/0.97) - 1$ used in the second calibration arguably implies too low of a price-dividend ratio relative to the data. This may in turn result in less accurate log-linear approximations compared to the internationally consistent calibration reported in the paper.

With the exception of the prespecified values for κ_1 and κ_1^d, the parameter values for Calibration 1 directly mirror the ones for the main calibration discussed in the paper. Otherwise, the key difference between the two calibrations relates to the persistence of the volatility-of-volatility processes, with Calibration 1 designed to match higher-frequency dynamic dependencies in the returns, and Calibration 2 better geared towards matching the intermediate one to six month dependencies.

Consistent with the calibration results reported in the paper, the results reported in Table A2 generally reveal a fairly close match between the model-implied and observed sample moments. The most notable differences between the two being the persistence of the consumption growth rate process and the average value of the VIX^2.

To further highlight the economic mechanisms and the way in which the differences in persistence manifest themselves in the two calibrations, following the work of Bollerslev *et al.* (2009) and Drechsler and Yaron (2011), the last three rows of the table report the R^2's from the return predictability regressions using the variance risk premium to predict the one to six months returns. Calibration 2 in particular, does a good job in terms of matching the corresponding R^2's reported in Bollerslev *et al.* (2009).

Further along these lines, each of the two calibrations imply predictive R^2's of 1.3% and 2.0%, respectively, for the P/D ratio at the longer one-year horizon, which closely match the 2.0% data value reported in Bansal and Yaron (2004). However, the relatively low value of the autocorrelation for the stochastic volatility process employed in both of the calibrations also means that they are not able to explain higher R^2's for the P/D ratio over even longer horizons. Meanwhile, increasing the autocorrelation of the stochastic volatility process from 0.76 to 0.98 (close to the 0.987 used by Bansal and Yaron (2004)) results in predictive R^2's for the P/D ratio at the 3 and 5-year horizons of 19.2% and 23.9%, respectively, both of which are closer to the data values of 19% and 37%, respectively, in Bansal and Yaron (2004), than are the calibrated values of 10% and 16%, respectively, reported therein. Of course, the longer multi-year horizon R^2's are much more difficult to reliably estimate in the data, so these additional calibration results should be carefully interpreted.

Table A1 Calibration Parameters

	Calibration 1		Calibration 2	
	Parameter	Source	Parameter	Source
Preferences				
ρ	$-\log(0.999)$	BYa	$-\log(0.999)$	BYa
γ	7.50	BYa	7.50	BYa
ψ	2.50	BYa	2.00	BYa
Linearization				
κ_1	$(1/0.997)-1$	BY	$(1/0.97)-1$	BST
κ_1^d	$(1/0.997)-1$	BY	$(1/0.97)-1$	BST
Consumption Growth				
μ_g	0.0015	BY	0.0015	BY
κ_x	$-\log(0.960)$	BYa	-log(0.979)	BY
σ_x	$0.044\sqrt{\mu_\sigma}$	BY	$0.044\sqrt{\mu_\sigma}$	BY
Dividend Growth				
μ_d	0.0013	BYa	0.0013	BYa
ϕ_x	3.0	BY	3.0	BY
ϕ_σ	$4.0\sqrt{0.55}$	BYa,K	$5.0\sqrt{0.60}$	BST,K
$\phi_{\sigma x}$	0.0	K	0.0	K
σ_d	$4.0\sqrt{0.45}\,\mu_\sigma^{1/2}$	BYa,K	$5.0\sqrt{0.40}\,\mu_\sigma^{1/2}$	BYa,K
Volatility				
μ_σ	0.0078^2	BY	0.0088^2	BST
κ_σ	$-\log(0.760)$	BST	$-\log(0.760)$	BST
μ_q	0.350×10^{-6}	BST	0.140×10^{-6}	BST
κ_q	$-\log(0.0001)$	BST	$-\log(0.60)$	BST
φ_q	0.07	BST	0.00679	BST

Note: BYa and BY denote small adjustments to, or values taken directly from, Bansal and Yaron (2004); BST denotes values calibrated by the authors; K denotes values from Kiku (2008).

Table A2 Calibration Results

	Observed	Source	Calibration 1	Calibration 2
System Dynamics				
Consumption†				
$\mathrm{E}(g_t)$	1.80	BY	1.80	1.80
$\sqrt{\mathrm{Var}(g_t)}$	2.93	BY	2.70	3.05
Dividends†				
$\mathrm{E}(d_t)$	1.54	DY	1.54	1.54
$\sqrt{\mathrm{Var}(d_t)}$	13.69	DY	10.81	15.24
Correlations†				
$\mathrm{Corr}(g_t, g_{t-1})$	0.43	DY	0.18	0.43
$\mathrm{Corr}(d_t, d_{t-1})$	0.14	DY	0.10	0.15
$\mathrm{Corr}(d_t, g_t)$	0.59	DY	0.74	0.77
Model Implied Moments				
Returns†				
$\mathrm{E}(r_{f,t})$	0.82	DY	1.19	1.37
$\sqrt{\mathrm{Var}(r_{f,t})}$	1.89	DY	4.43	3.23
$\mathrm{E}(r_{m,t} - r_{f,t})$	6.23	DY	5.95	6.19
$\sqrt{\mathrm{Var}(r_{m,t})}$	19.37	DY	19.17	22.24
Volatility and Premium				
$\mathrm{E}(\mathrm{VIX}^2)$	32.81	BST	37.87	47.69
$\mathrm{E}(vp_t)$	8.96	BST	7.25	6.48
Predictability Regressions				
R^2(1 Month)	1.07%	BTZ	0.75%	4.70%
R^2(3 Months)	6.82%	BTZ	0.28%	6.76%
R^2(6 Months)	5.42%	BTZ	0.16%	5.55%

Note: † indicates the reported value is appropriately annualized to ease interpretation. $\mathrm{E}(r_{m,t} - r_{f,t})$ and $\sqrt{\mathrm{Var}(r_{m,t})}$ refer to the equity premium and the equity return standard deviation, respectively. For sources, BST denotes values reported in Table **??** above; BTZ comes from Bollerslev *et al.* (2009); BY denotes values taken from Bansal and Yaron (2004); DY denotes values from Drechsler and Yaron (2011).

References

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